Tutorial 6. Oct. 19.

~~1. Let γ be a smooth~~

1. (a). Evaluate the integrals

$$\int_\gamma z^n dz.$$

for all integers n. Here γ is any circle centered at the origine with the positive orientation

Note: By Positive orientation it Means $\circlearrowleft$.

(b). Same question as before, but with γ any circle not containing the origin.

(c). Show that if $|a| < r < |b|$, then

$$\int_\gamma \frac{1}{(z-a)(z-b)} dz = \frac{2\pi i}{a-b}.$$

where γ denotes the circle centered at the origin, of radius r, with the positive orientation.

2. Let γ_R be the rectangle with vertices R, $R+iS$, $-R+iS$, $-R$. and positive oriantation. Let $f(z) = e^{-\pi z^2}$

Let $I(R) = \int_0^S f(R+iy)\, i\, dy = \int_0^S e^{-\pi(R^2 + 2iRy - y^2)}\, i\, dy.$

Show $I(R) \to 0$ as $R \to +\infty$.

3. Show $1 = \int_{-\infty}^{+\infty} e^{-\pi x^2} dx.$ Hint: Calculate $\left(\int_{-\infty}^{+\infty} e^{-\pi x^2} dx\right)^2$

4. show $e^{-\pi \xi^2} = \int_{-\infty}^{+\infty} e^{-\pi x^2} e^{-2\pi i x \xi} dx.$